IDENTIFYING THE MARGINAL BORROWER UNDER ADVERSE SELECTION

Abstract. What are the distributional implications of adverse selection in credit markets? We present a simple model of financial intermediation between a monopolist lender and borrowers with private information. To specifically examine heterogeneity in credit outcomes, we introduce different groups of borrowers with varying degrees of adverse selection. We have three results. First, we show that greater adverse selection can cause the lender's marginal revenue curve to be more elastic. Second, groups with greater adverse selection—even though they are a worse credit risk—may receive more loans than other groups. Finally, we show that following a fall in the cost of bank lending, consumer groups with greater adverse selection may receive a larger increase in credit than other groups but not necessarily.

Keywords: Credit markets, inequality, adverse selection, marginal borrower
JEL Codes: D11, G21, G28, 016

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1. Introduction

The effect of adverse selection in credit markets has attracted substantial academic and policy attention. Moreover, given that poorer or minority populations may be more exposed to the problems of adverse selection in credit markets, there has been increasing empirical and policy attention on the distributional effect of changes in credit policy (Agarwal et al. [2018] and Choudhary and Jain [2022]). This paper answers an important complementary, but understudied, theoretical question: If the cost of lending decreases, which borrowers receive relatively more credit and how does this effect depend on adverse selection?

This paper fills the gap in the theoretical literature by introducing a model of financial intermediation between a monopolist lender and credit-constrained consumers with private information. To specifically examine the consequences of adverse selection—defined as the outcome that as interest rates rise, the proportion of riskier borrowers willing to originate a loan also rises—we model different groups of consumers that vary in their proportion of creditworthy borrowers. We start by modelling adverse selection using a linear function that demonstrates the key considerations and then extend this model to a more general function.

Our paper's main result is that as we decrease the lender's funding cost, groups with a greater problem of adverse selection (and no other differences across groups) can receive relatively *larger* increase in lending on the extensive margin than other groups because the lender's marginal revenue curve can be more elastic for these groups. Our result contradicts Agarwal et al. [2018] informal argument that those groups that are characterized by greater adverse selection will benefit less from a decrease in the lender's cost of funding. Intuitively, as the lender's funding cost falls, the lender will expand lending by reducing interest rates. The magnitude of this lending expansion for each consumer group will depend on the slope of the lender's marginal revenue curve, which we show is *more* elastic as the degree of adverse selection increases for most lending.

Our model is motivated by consumer lending where borrowers often have public but not perfect signals of their creditworthiness. For example, consumers in many countries are screened using credit scores. In these situations, lenders will categorize consumers using the public information into different groups. However, due to the signals being imperfect, there will be variation in credit outcomes within each group. Moreover, we may expect that the problem of adverse selection is larger for consumers with lower credit scores because there is less available credit information thereby increasing the variation in credit quality.

To explore the relative effect of changes in a lender's funding cost on the marginal borrower and heterogeneous effects across borrowers, we modify Stiglitz and Weiss [1981] to allow groups of borrowers to vary on three dimensions. First, we allow the fraction of creditworthy borrowers in each group to vary causing the degree of adverse selection to vary across groups. Second, we allow the return of the consumer's project to vary across groups; and finally, we allow the lender's marginal cost of lending to vary across groups. Moreover, within each group, each consumer has a specific type that determines both the consumer's return conditional on success, and the consumer's likelihood of success. To introduce the concept of adverse selection, this consumer type is private information and not visible to the lender.

The seminal paper by Agarwal et al. [2018] empirically and theoretically explore the heterogeneous effect on credit following a monetary expansion. Agarwal et al. [2018] informally argue that groups that suffer more from adverse selection have more inelastic marginal profit curves. Our model—which endogenously introduces adverse selection—finds a different result. The key modelling difference between our papers is that we allow the lender to alter the interest rate and consequently adjust total lending, whereas Agarwal et al. [2018] consider adjustment on only the quantity margin but not interest rates. This difference in assumptions has a substantive effect on the model's inferences. Using this assumption, Agarwal et al. [2018] descriptively argue that the problem of adverse selection leads to the lender's marginal profitably curve to be more inelastic for those groups where the problem of adverse selection is largest, and in turn, this leads the increase in lending to be the smallest for the groups with the largest degree of adverse selection.[1] In contrast, in our simple linear model, we find that the lender's marginal profitability curve is most elastic for those groups where the problem of adverse selection is largest.[2]

In our model with adverse selection, if the lender wishes to increase lending, the lender must reduce the interest rate to attract additional borrowers. Therefore, in an environment with

[1] Our model concentrates on the marginal revenue curve (rather than the marginal profitability curve as in Agarwal et al. [2018]) but given that we assume that the lender's marginal cost curve is constant, the marginal profitability curve and the marginal revenue curve will have the same slope.

[2] In our more general model, we find that the lender's marginal revenue curve is more inelastic for groups with more adverse selection for most parameter values. That is, there are some extreme values where the lender's curve is less elastic for the group with more adverse selection.

adverse selection, a reduction in the interest rate prompts a *virtuous* circle because the lower interest rate induces safer borrowers to take loans allowing the lender to reduce interest rates even further. Whereas the model in Agarwal et al. [2018] assumes that riskier borrowers are more credit-constrained and consequently lenders can adjust the quantity of lending without changing the interest rate. Consistent with our model's assumptions, there is substantial empirical support demonstrating that higher interest rates attract a riskier composition of borrowers. For instance, Ausubel [1999] and Agarwal et al. [2010] using a quasi-experimental approach show that worse credit-risk borrowers are relatively more attracted to credit card solicitations with higher interest rates.

Our parsimonious simple model allows for differences in loan demand across groups by using a reduced-form elasticity parameter. Therefore, we can account that for some groups—say rich individuals who are looking to renovate their home—may be highly interest rate sensitive. Whereas others may be significantly less sensitive—say poorer individuals who are cashflow constrained. Therefore, a secondary result in our paper is that the total change in lending depends on more than the degree of adverse selection in that group. Specifically, those consumer groups with a higher elasticity of demand with respect to interest rates are more responsive to changes in the lender's funding cost because the lender's marginal revenue curve is more elastic for these groups. This result ensures that empirical predictions of our model require knowledge of both the elasticity of demand for each consumer group as well as the extent of adverse selection in each group.

The model presented in our paper is a stylized example of consumer lending that analyzes the distributional implications of adverse selection on the extensive margin. To focus on this channel, we intentionally omit other key attributes of consumer lending (such as moral hazard or strategic default), and only model one form of credit lending—a single lender with market power. Moreover, a key intentional omission is a discussion of changes in lending along the intensive margin—that is, how does adverse selection affect the size of loans in responsive to lower funding costs. This simplification does abstract away from practiced loan mechanisms to counteract adverse selection. For instance, groups with greater adverse selection may be subject to more binding loan conditions that restrict loan size such as loan-to-value ratios, debt-to-income limits, or credit limits based of credit scores. Therefore, even if interest rates fall, these groups may be unable to expand their credit usage.

Our paper builds on the large theoretical literature on adverse selection in credit markets starting from Jaffee and Russell [1976] and Stiglitz and Weiss [1981]. We present a simplified version of the model in Stiglitz and Weiss [1981] and we model adverse selection in a similar way with riskier borrowers attracted at higher interest rates. The key addition to a standard models of adverse selection is that we allow the degree of adverse selection within a group of borrowers to vary. This additional model element allows us to compare how different groups of borrowers are affected by changes in a bank's funding cost.

Understanding the importance of who the marginal borrower is a crucial question for social welfare and policymakers with greater research and policy attention on how changes in monetary policy may have heterogeneous effects for both consumers and firms. Di Maggio et al. [2017] show large heterogeneous responses to lower mortgage payments (using variation in the timing of mortgage rate resets) with low income borrowers substantially increasing consumption and increasing household leverage following a large positive income shock. Gerardi et al. [2020] show that minorities with mortgages benefit the least following lower mortgage interest rates because they are less likely to refinance their mortgage. Heider et al. [2019] shows that banks that were more adversely affected by falling negative interest rates lent less overall and relatively more to risky borrowers. Our paper contributes to this literature by describing the theoretical determinants of who the marginal borrow is.

Section (2) presents our simple model, section (3) extends the model by presenting results from a more general model, and section (4) provides concluding thoughts.

2. Model

There are multiple different consumer groups (indexed by the letter g), each of measure one. Every consumer (indexed by the letter i) has a project that requires funding. There is a single bank that can finance these projects. The value of the project and the expected success of the project is private information to the consumer. However, the consumer's group is public information. Therefore, the bank can make group-specific interest rates.

2.1. **Consumer's problem.**

Consumers have limited liability, and will only repay the loan if their project is successful. This allows consumers to choose loans which are socially inefficient—that is, consumers may choose to invest in projects that have a lower expected return than the bank's funding costs.

The return conditional on success and probability of success for consumer i in group g's is modelled by two parameters, $\theta_{i,g}$ and a_g. $\theta_{i,g}$ is within group variation, is uniformly distributed between zero and one, private information to the consumer, and is neither observable or verifiable by the bank. a_g is the degree of adverse selection within each group and is discussed in more detail later.

A consumer of type i in group g, with a successful project will earn a gross return or payoff of $(\theta_{i,g}/b_g)$ and an unsuccessful project will earn zero. For convenience, we normalize the utility of consumers who do not take a loan to be zero. Therefore, our model ensures that, conditional on project success, those consumers with a higher θ_i have higher gross earnings.

A consumer of type i in group g with a loan at interest rate R_g will have the following utility if the loan is successful (note since the consumer's group is public information, the bank can offer different interest rates to each group):

$$U_{i,g}(\theta_{i,g}, R_g) = \frac{\theta_{i,g}}{b_g} - R_g$$

where, b_g is a parameter that measures the interest rate elasticity of demand (and restricted to be strictly positive), and can vary across groups. A higher b_g ensures that the consumer's demand for a loan is more sensitive to rises in the interest rate. Empirically, you would expect that the interest rate elasticity of demand, proxied by b, is likely highly correlated with the both the income level of the borrower and the level of adverse selection within the group. For instance, higher income borrowers may take out loans for luxuries, and as such are highly sensitive to the interest rate. Whereas, low income borrowers demand for loans may be highly inelastic due to cashflow constraints.[3]

We define an individual's project's probability of success, $p(\theta_{i,g}, a_g)$, as a function of $\theta_{i,g}$ and adverse selection, a_g, as:

$$\text{Pr}(\text{Success}|\theta_i, a_g) \quad = \quad p(\theta_{i,g}, a_g) = \bar{p}_g + \left(\frac{1}{2} - \theta_{i,g}\right) a_g \tag{1}$$

[3]We thank an anonymous referee for this additional interpretation of how to consider the parameter b_g.

where, a_g is a parameter that determines the extent of adverse selection within each consumer group; and, $\bar{p}_g$ is the average repayment probability and varies across groups.[4] We make two assumptions on the parameter space of p_g and a_g. First, p_g is sufficiently high by assuming that $\bar{p}_g$ is between $1/2$ and 1, and second, that a_g is between 0 and p_g. Together, these two assumptions ensure that the probability of success is within the range of 0 and 1.

2.2. **Key features of the functional form.** Our functional form choice for the probability of success exhibits three key features.

Feature 1: Borrowers with higher θ_i have projects that are less likely to succeed (equation 2). Therefore, agents with higher $\theta_{i,g}$ have projects – if successful – earn more, but are less likely to succeed. This assumption is crucial for ensuring within group adverse selection as when interest rates rise, individuals with riskier projects will take loans.

$$\frac{\partial p(\theta_{i,g}, a_g)}{\partial \theta_i} < 0 \tag{2}$$

Feature 2: The average probability of success within a group is independent of the degree of adverse selection within a group, varies across groups, and is equal to $\bar{p}_g$ (equation (3)). Therefore, we can have two groups with different degrees of adverse selection (a_g) but the same expected probability of success ($\bar{p}_g$).

$$\begin{aligned} E_g(success) &= E[p(\theta_{i,g}, a_g)] \\ &= E[\bar{p}_g + \left(\frac{1}{2} - \theta_{i,g}\right) a_g] \\ &= \bar{p}_g + \left(\frac{1}{2} - E[\theta_{i,g}]\right) a_g \\ &= \bar{p}_g \end{aligned} \tag{3}$$

Feature 3: The variability of the probability of success within a group is larger for groups with higher a_g (equation 4). Therefore, we define groups with greater variability of success as having a larger degree of adverse selection. Note that if a_g is equal to zero, then there is

[4]In section (3), we extend this function into a more general function.

no variability within group and consequently no adverse selection.

$$\begin{aligned} Var_g(success) &= Var[p(\theta_{i,g}, a_g)] \\ &= Var[\bar{p}_g + \left(\frac{1}{2} - \theta_{i,g}\right) a_g] \\ &= a_g^2 Var[\theta_{i,g}] \end{aligned} \tag{4}$$

2.3. **Bank's problem.** The lender sets an interest rate R_g for each group. The cost of a bank making a loan to a consumer in group g is $\kappa_g c$, where κ_g and c are both greater than zero and κ_g can vary across consumer groups. For instance, the cost of servicing rural consumers may be higher than for urban consumers due to transport or operational costs.

2.4. **Timing.**

(1) Nature draws a consumer's type ($\theta_{i,g}$)
(2) Consumer decides whether to ask the bank for a loan.
(3) Bank offers an interest rate R_g to the consumer.
(4) Consumer decides whether to accept or reject the bank's offer.
(5) If the consumer accepts the bank's offer, the project is undertaken.
(6) Game ends and payoffs are realized.

2.5. **Equilibrium.** We assume only those agents who could earn a strictly positive rent take a loan.[5] Therefore, the demand function for loans by consumers of type g is:

$$D_g(R_g) = \int 1_{(U_{i,g}(\theta_{i,g}, R_g) > 0)} f(\theta) d\theta \tag{5}$$

$$= \int 1_{(\theta_{i,g}/b_g - R_g > 0)} d\theta \tag{6}$$

$$= 1 - b_g R_g \tag{7}$$

Equation (5) follows from only those consumers who expect a positive utility if their loan succeeds will take a loan. Equations (6) and (7) follow from substituting the uniform distribution of θ into equation (5).

[5]This is an innocuous assumption because a simple model extension would be to require consumers to pay a tiny cost to apply for loans.

The bank's expected profit from offering loans at an interest rate R_g to group g is:

$$\pi_g(R_g) = D_g(R_g).R_g.\tilde{p}_g(R_g) - \kappa_g c D_g(R_g) \tag{8}$$

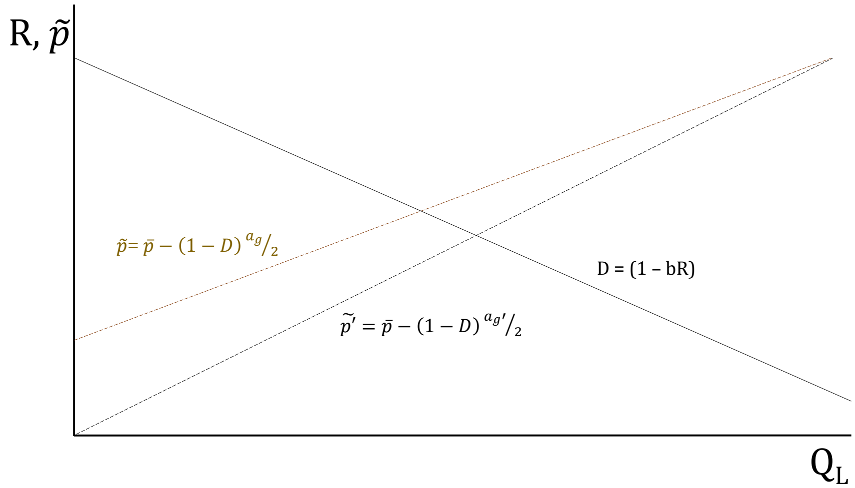
where $\kappa_g c$ is the bank's marginal (and average) cost for offering a loan to a consumer in group g. $\tilde{p}_g(R_g)$ is the expected repayment probability from lending to group g if the bank charges an interest rate R_g, which equals:

$$\begin{aligned}
\tilde{p}_g(R_g) &= \frac{1}{1-F(\theta|\theta_{i,g}/b_g - R_g > 0)} \int p(\theta_{i,,g}|\theta_{i,g}/b_g - R_g > 0) f(\theta)\, d\theta \qquad (9)\\
&= \frac{1}{1-b_g R_g} \int_{bR}^{1} \bar{p} + \left(\frac{1}{2} - \theta_{i,g}\right) a_g \, d\theta \\
&= \bar{p} - \frac{a_g}{2} b_g R_g = \frac{a_g}{2} D_g(R_g) + \bar{p} - \frac{a_g}{2} \\
&= \bar{p} - \frac{(1-D_g)}{2} a_g \qquad (10)
\end{aligned}$$

Figure (1) shows the relationship between the degree of adverse selection and the average repayment probability: Raising the degree of adverse selection within a group causes the average repayment probability to weakly fall for all positive levels of demand. Or more formally, partially differentiating the average repayment probability (equation (10)) with respect to adverse selection:

$$\frac{\partial \tilde{p}_g(R_g)}{\partial a_g} = \frac{1-D_g}{2} \geq 0 \tag{11}$$

FIGURE 1. The relationship between the degree of adverse selection, a_g and the average average repayment probability, $\tilde{p}$.



This figure first shows that the demand function and the average repayment probability, $\tilde{p}$, are closely related. For higher interest rates, only the riskiest borrowers are willing to borrow causing both demand and the average repayment probability to be low. As the interest rate declines, relatively less-risky borrowers are attracted to take loans causing the average repayment probability and total demand to rise. Second, this figures illustrates the effect of increasing the degree of adverse selection, a_g to a'_g, on the average repayment probability: A higher a_g causes the average repayment probability to be lower for all values of demand (as seen by comparing the $\tilde{p}'$ curve to the $\tilde{p}$ curve).

Substituting equations (7) and (10) into equation (8), the bank's profit problem for group g becomes:

$$E[\pi_g(D_g)] \quad = \quad \max_{D\in[0,1]} D_g \left[\frac{1}{b_g}(1-D_g)\left(\frac{a_g}{2}D_g + \bar{p}_g - \frac{a_g}{2}\right) - \kappa_g c\right] \tag{12}$$

The bank will maximize expected profits by adjusting the interest rate, and thereby demand. Assuming the bank's optimal lending is non-zero, the bank's optimal interest rate will set the marginal revenue of lending equal to the marginal cost of lending. Figure (2) and (3) shows the marginal revenue curve with and without adverse selection, respectively.

$$\text{Expected marginal revenue for group } g = \text{Marginal cost for group } g$$

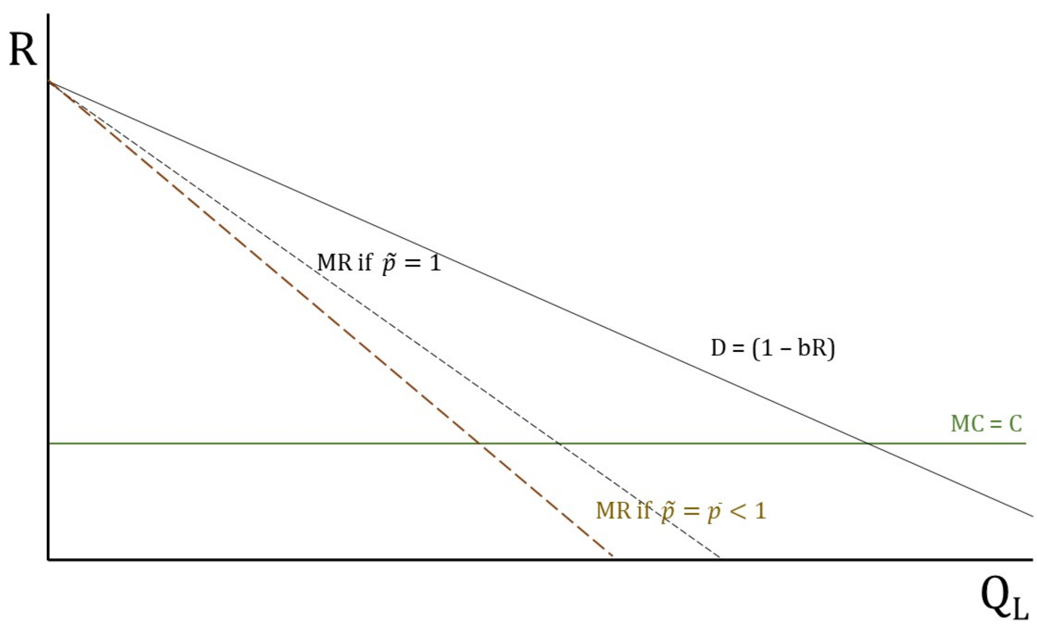
$$\frac{\partial E[Revenue(D_g)]}{\partial D_g} = \frac{\partial C_g(D_g)}{\partial D_g}$$

$$\frac{1}{2b_g}\left[2\bar{p}_g - a_g - 4(\bar{p}_g - a_g)D_g^* - 3a_g D_g^{*2}\right] = \kappa_g c \tag{13}$$

If there is no adverse selection (that is, $a_g = 0$) then equation (13) simplifies to the following and the marginal revenue curve becomes a linear decreasing function in demand:

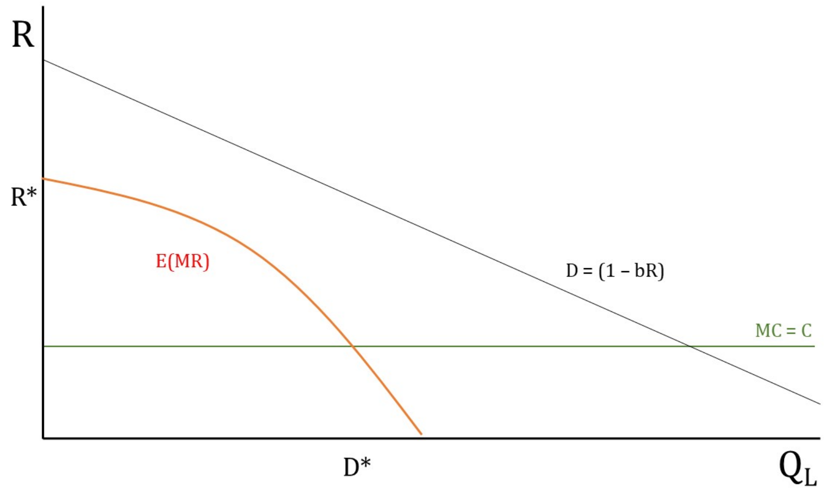
$$\frac{1}{b_g}\bar{p}_g(1 - 2D_g^*) = \kappa_g c \quad \text{if } a_g = 0 \tag{14}$$

Figure 2. The lender's marginal revenue curve with no adverse selection.



This figure shows the bank's expected marginal revenue curve under two conditions. First, when $\tilde{p} = 1$, therefore when all consumers repay their loan. Second, when there is no adverse selection ($a_g = 0$) and some loan defaults, so $\bar{p} = \tilde{p} < 1$. In both cases, the lender's marginal revenue is a linear downward sloping curve and is similar to a monopolist serving a market with linear demand.

FIGURE 3. The effect of adverse selection on the expected marginal revenue curve.



This figure shows how the bank's expected marginal revenue curve is affected by adverse selection. As the bank initially lowers the interest rate, the bank's expected marginal revenue curve decreases slowly. As discussed in detail in section (2.6), as the bank lowers the interest rate there are three effects. First, the bank receives the additional revenue from providing more loans. Second, the bank receives less revenue from all the inframarginal borrowers due to the lower interest rate. Finally, the bank benefits from the converse of the adverse selection effect (advantageous selection), whereby the marginal borrower is more creditworthy, thereby raising the average probability of being repaid. As the bank reduces interest rates, the second factor becomes larger causing the slope of the marginal revenue curve to become steeper.

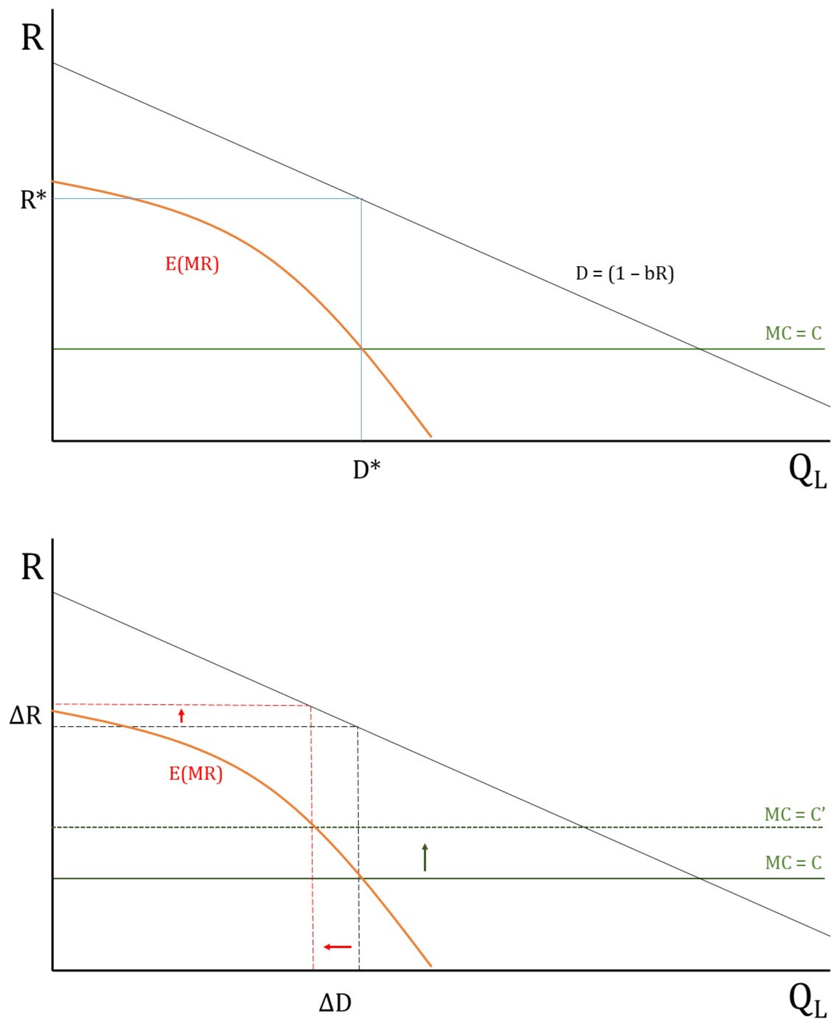
Turning to the equilibrium interest rates and quantity of loans for each group. Using equations (7), (8), and (13) we can solve for the bank's optimal provision of loans $D_g^*(a_g)$ and interest rate $R_g^*(a_g)$ for group g:

$$D_g^*(a_g) = \begin{cases} \frac{1}{2} - \frac{b_g \kappa_g c}{2\bar{p}_g}, & \text{if } \pi_g(R_g^*, D_g^*) > 0 \;\&\; a_g = 0 \\ \frac{1}{3a_g}\left[2a_g - 2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p} - 6a_g b_g \kappa_g c}\right], & \text{if } \pi_g(R_g^*, D_g^*) > 0 \;\&\; a_g > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$R_g^*(a_g) = \begin{cases} \frac{1}{2b_g} + \frac{\kappa_g c}{2\bar{p}_g}, & \text{if } \pi_g(R_g^*, D_g^*) > 0 \;\&\; a_g = 0 \\ \frac{1}{3a_g b_g}\left[a_g + 2\bar{p}_g - \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p} - 6a_g b_g \kappa_g c}\right], & \text{if } \pi_g(R_g^*, D_g^*) > 0 \;\&\; a_g > 0 \end{cases}$$

Figure (4)'s top panel shows how the equilibrium interest rate and total lending to borrowers in group g are determined by the intersection of the marginal cost and marginal revenue curve for an interior solution. The bottom panel shows the equilibrium changes in the interest rate and lending following an increase in the bank's marginal cost. If the bank's funding cost increases, the bank will charge a higher interest rate and will reduce the number of loans issued.

FIGURE 4. The effect of a change in a bank's funding cost on interest rates and lending



The top panel shows the equilibrium interest rate and quantity of loans. The bottom panel shows how the equilibrium interest rate and the quantity of loans respond to an increase in the bank's cost of funding. If the bank's cost of funding rises, the bank's marginal cost curve shift upwards, causing a lower equilibrium demand and higher interest rate.

2.6. **The effect of adverse selection on the lender's marginal revenue curve.** Given that the lender's optimal lending strategy is where marginal revenue is equal to marginal cost (for an interior solution), this section analyses in detail how adverse selection affects the lender's marginal revenue curve. In figure (5) we present an example with three different levels of adverse selection: zero (blue), medium (red), and high (yellow). The panel plots the lender's optimal interest rate for a given level of loan supply (top-left panel), the average repayment probability, $\tilde{p}$ (top-right panel), the lender's average revenue per loan (bottom-left) and the lender's marginal revenue (bottom-right panel).

In the top-left panel, there is no effect from adverse selection because loan demand is independent of adverse selection for a given interest rate (equation 7).

In the top-right panel, the lender's average repayment probability is a constant for the group with zero adverse selection. Whereas, for the other two groups, average repayment is increasing in loan supply because as the lender reduces interest rates, the lender attracts safer borrowers (equation 10).

In the bottom-left panel, the lender's expected average revenue—the product of the interest rate and the average repayment probability—is always greater for the group with less adverse selection for a given level of supply.

In the bottom-right panel, the marginal revenue curve varies across these three groups with the three marginal revenue curves intersecting and having different slopes.

How does the marginal revenue curve depend on the extent of adverse selection? Interestingly, adverse selection causes the slope (lemma 1)) and level (lemma 2) of the marginal revenue curve to change. Moreover, for some levels of supply, the marginal revenue is greater for higher degrees of adverse selection. We state the lemmas and then describe the intuition.

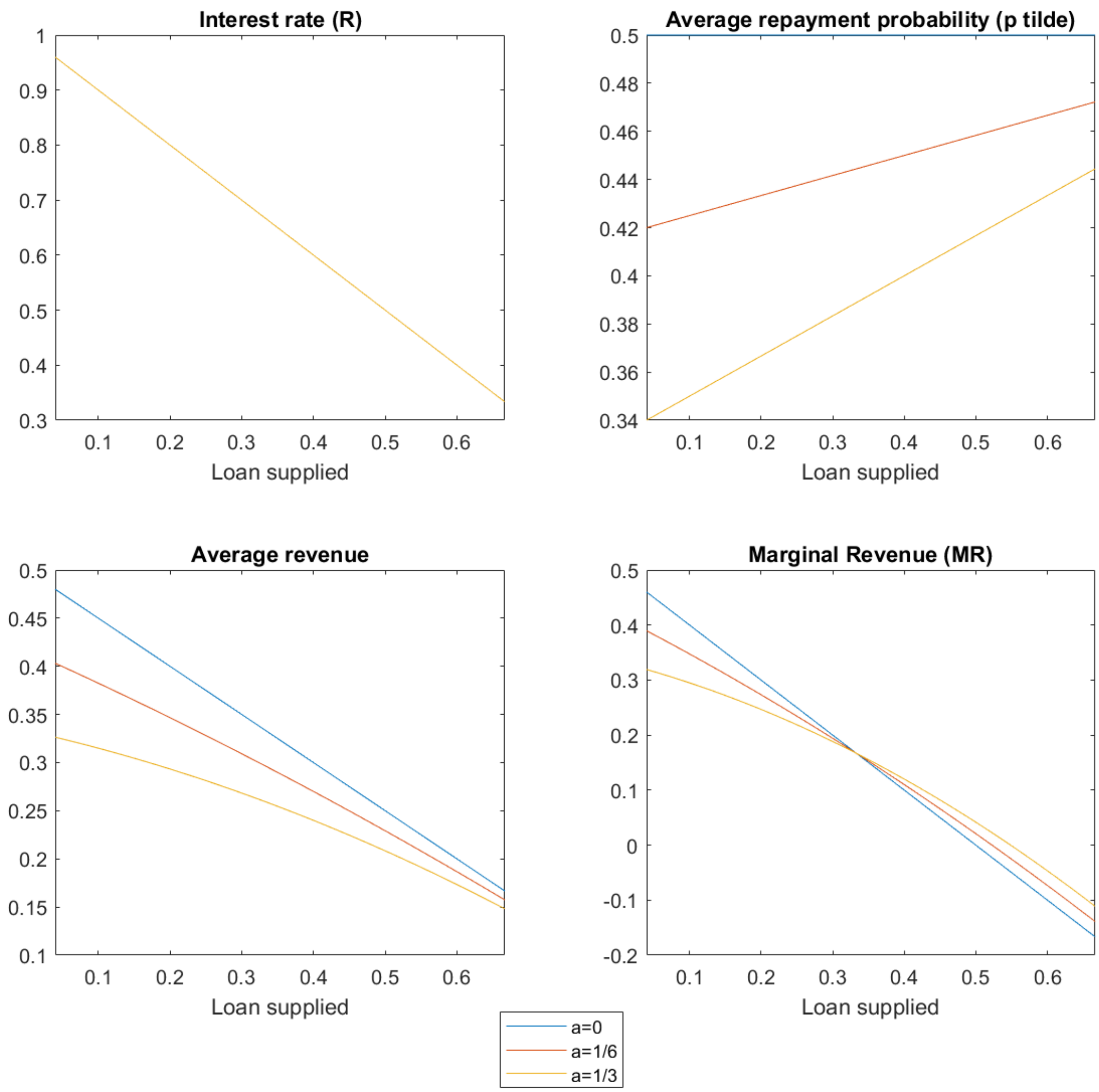
Lemma 1. ***The lender's marginal revenue is more sensitive—more elastic—to changes in loans supplied if adverse selection is higher for a given level of loans supplied.*** *Formally,*

$$\frac{d^2E[MR]}{da_g dD_g} \quad \geq \quad 0 \quad \forall\, D_g \in [0, 2/3] \tag{15}$$

Therefore, equation 31 states that the lender's marginal revenue decreases faster for small changes in supply when adverse selection is lower.[6]

[6]Note that the maximum quantity supplied in equilibrium is also less than or equal to 2/3, proof is in Appendix A1, therefore equation 31 holds for all possible values of loan supply.

FIGURE 5. The effect of adverse selection on the interest rate, the average repayment probability, average revenue, and marginal revenue.



This figure shows the relationship between the interest rate, the average repayment rate ($\tilde{p}$), average revenue per loan, and marginal revenue for three different levels of adverse selection (a_g). The blue curve represents no adverse selection ($a_g = 0$), the red curve represents medium adverse selection ($a_g = 1/3$), and the yellow curve represents high adverse selection ($a_g = 2/3$)

Lemma 2. ***The lender's marginal revenue curve is greater for groups with less adverse selection at low values of supply.*** *Consider two consumer groups, g and g', that have identical demand elasticities and average repayment probabilities but one group, group g', has a greater degree of adverse selection within the group (that is, $a_{g'}$ is strictly* *greater than a_g). Then the lender's marginal revenue curve for the group with a lower adverse selection is weakly greater for low supply and weakly lower for large supply.*

To demonstrate this result, we differentiate the marginal revenue curve with respect to the degree of adverse selection (a_g).

$$\frac{dE[MR]}{da_g} = \frac{1}{2b_g}(-1 + 4D_g - 3D_g^2)$$

$$\frac{dE[MR]}{da_g} = \begin{cases} \leq 0, & \text{if } 0 \leq D_g \leq 1/3 \\ \geq 0, & \text{if } D_g \geq 1/3 \end{cases} \tag{16}$$

Therefore, equation (16) states that a higher degree of adverse selection will cause the marginal revenue curve to be lower at low levels of supply (supply less than 1/3).

The bottom right panel in figure (5) illustrates lemmas (1) and lemma (2). Specifically, for supply less than 1/3, groups with less adverse selection have greater marginal revenue (lemma 2). Moreover, as the lender increases supply, then the marginal revenue curve decreases faster for the group with less adverse selection (lemma (1)).

A corollary of lemma (2) is that the lender will give *more* loans to groups with greater adverse selection if the marginal cost of lending is low. Whereas, if marginal costs are high, the lender will give more loans to groups with less adverse selection.

To analyze why the expected marginal revenue curve is affected by adverse selection, we can write the expected marginal revenue function into three components:

$$E(MR) = \frac{dE[Revenue(D_g)]}{dD_g} = \frac{d\,[D_g.R_g(D_g).\tilde{p}_g(D_g)]}{dD_g}$$

$$= \underbrace{R_g\tilde{p}_g}_{\text{(i) Mean revenue from one more borrower}} + \underbrace{\frac{dR_g}{dD_g}D_g\tilde{p}_g}_{\text{(ii) Reduction in rev. from inframarginal borrowers}} \tag{17}$$

$$+ \underbrace{\frac{d\tilde{p}_g}{dD_g}(R_gD_g)}_{\text{(iii) Increase in rev. from increase in average repayment probability}}$$

Therefore, (i) $R_g\tilde{p}_g$ is the direct effect on revenue from increased lending: (ii), $\frac{dR_g}{dD_g}D_g\tilde{p}_g$ is the effect from lower revenue from all inframarginal borrowers; and, (iii), $\frac{d\tilde{p}_g}{dD_g}(R_gD_g)$ is the increase in revenue from the increase in the average repayment probability and is caused by the positive selection effect from reducing interest rates.

The effect of the first two components is similar to when a mononpolist increases output by one unit; that is, the lender gains from revenue from new customers and the lender losses revenue from all the inframarginal customers. Moreover, the effect of adverse selection is indirectly included in these two effects through the inclusion of the variable, $\tilde{p}_g$. Turning to the final component, this captures the increase in revenue caused by an increase in the average repayment probability (because a lower interest rate attracts more creditworthy borrowers).

Using this simple model, we can analyze how a change in a bank's funding cost affects the bank's lending decisions.

Proposition 1. *Consider an interior solution and two consumer groups, g and g', that solely differ on the problem of adverse selection within the group (specifically , $a_{g'}$ is strictly greater than a_g, and both groups have the same demand elasticities, b_g, same average repayment, $\bar{p}_g$, and same marginal cost of lending, κ_g).* ***Then, the group with the larger degree of adverse selection will have a larger increase in lending if the bank's funding cost falls.***

The proof of this proposition follows from differentiating the demand function with respect to the lender's cost of funding, c. For completeness, the proof is in the Appendix A2.

The intuition for this results follow from lemma (1). As the lender's cost of capital falls the marginal revenue curve is more elastic for the group with higher adverse selection. Therefore, this effect ensures the group with more adverse selection receives the relatively *larger* increase in credit.

3. Extension: Generalizing the model

To examine whether the results in the previous section are robust to different function forms, this section considers a more general function of adverse selection. However, this more general function does not provide closed-form solutions for total lending and consequently limits some of the analysis. Ultimately, in this more general model, we show that changes in lenders' funding costs *may* transmit more strongly to adversely selected pools of borrowers but not *necessarily*. That is, we show that for low-to-moderate amount of loans supplied, the results of the previous section hold; but, if lending is sufficiently high, there are some parameters where Proposition (1) no longer holds.[7]

This section starts by outlining the more general function for adverse selection (where the function described in equation (1) in section (2) is a special case). We describe the key features of this more general function and explain how this functional form retains the three key features described in section (2.2). Second, we describe how this more general function affects both the average repayment rate and the expected marginal revenue curve for different levels of loan supply. Finally, we restate a revised Proposition (1) for this more general function.

Consider the more general function for the probability of a project success for individual i in group g:

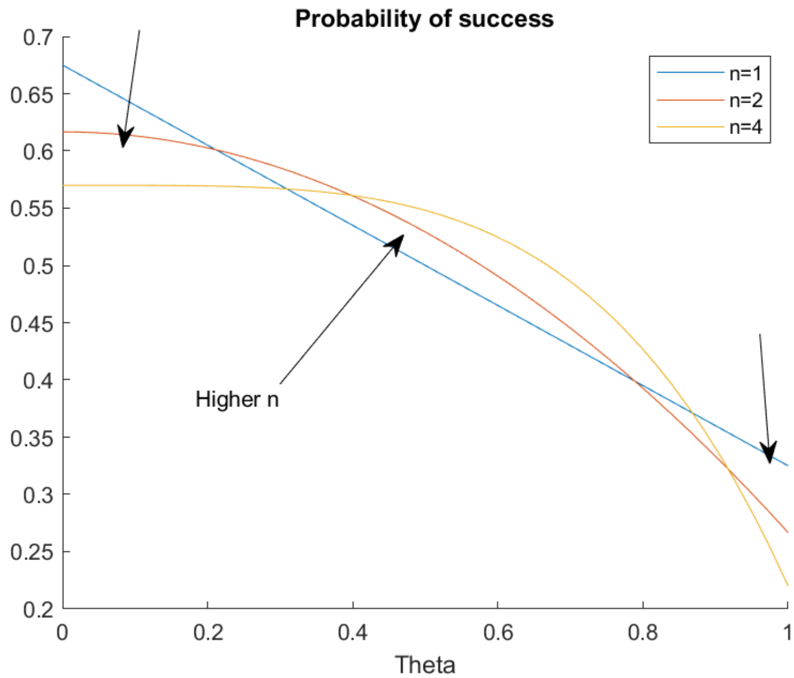
$$\Pr(\text{Success}|\theta_i, a_g, n) \quad = \quad p(\theta_{i,g}, a_g, n) = \bar{p}_g + \left(\frac{1}{n+1} - (\theta_{i,g})^n\right) a_g \tag{18}$$

with n being a positive number larger than or equal to 1. In this specification, the parameter n can be considered the degree at which adverse selection in the population changes. Moreover, note that the model analyzed in section (2) is just a special case of this model where n is set to 1 and consequently the rate at which adverse selection in the population changes is a constant rate.[8]

To build intuition for n alters the probability of a project's success, figure (6) plots the probability of success for three different levels of n, and holding all other parameters fixed. A higher n causes the probability of a project success to be significantly lower for individuals close to the ends of the distribution, whereas, those in the middle of the distribution have higher probabilities of success.

[7]For ease of exposition, we assume the parameters b and κ from the earlier section are equal to 1.
[8]With this more general function, we require the additional restriction that $\bar{p} \leq 1 - a_g/n+1$ to ensure that the probability of success is bounded below one.

FIGURE 6. The probability of success (p) for different degrees of adverse selection in the population (n)



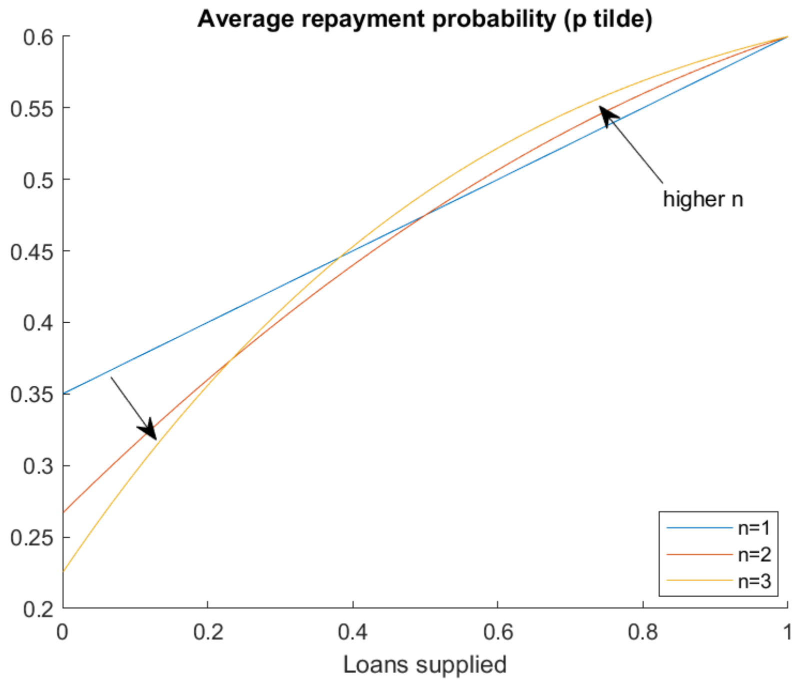
This figure shows the relationship between the probability of success (p) and θ_i as we adjust the degree of how adverse selection changes within the population, n. The black arrows demonstrate how the curves change as we increase n. A higher n ensures that the probability of success is lower for individuals close to the ends of the distribution and higher success probability for those close to the middle of the distribution.

To further investigate how changes in n affect lending, figure (7) shows the average repayment rate curve ($\tilde{p}$) for different levels of n, while holding all other parameters fixed.[9] A higher n ensures that the average repayment rate is both more elastic and lower at low levels of loan supply (in figure (7 the slope and level of the yellow curve is greater than the blue line at low levels of loans supplied). Whereas, a higher n ensures that the average repayment rate curve is both relatively insensitive and higher at high levels of loan supply.

The key advantages of this functional form for adverse selection mirror the three features described in Section (2.2). First, borrowers with higher θ_i have projects that are less likely to succeed, holding all other parameters constant (feature 1). Second, the average probability of success within a group is independent of a_g; specifically, the probability is equal to $\bar{p}_g$ for

[9]Recall from equation (9) that $\tilde{p} = \frac{1}{1-F(\theta_{i,g}|\theta_{i,g}/b_g - R_g > 0)} \int p(\theta_{i,g}|\frac{\theta_{i,g}}{b_g} - R_g > 0) f(\theta) d\theta$, that is, the mean repayment rate for the lender from the set of people that take a loan.

Figure 7. The relationship between the average repayment rate ($\tilde{p}$) and the degree at which adverse selection in the population changes (n)



This figure shows the relationship between the average repayment rate ($\tilde{p}$) and the loans supplied as we adjust the degree of how adverse selection changes within the population, n. The black arrows demonstrate how the curves change as we increase n. A higher n ensures that the average repayment rate is more sensitive at low levels of loan supply but is relatively insensitive at high levels of loan supply.

each group (feature 2).[10] Finally, the variance of success within a group is larger for groups with a higher degree of adverse selection (a_g), holding all other parameters constant (feature 3).

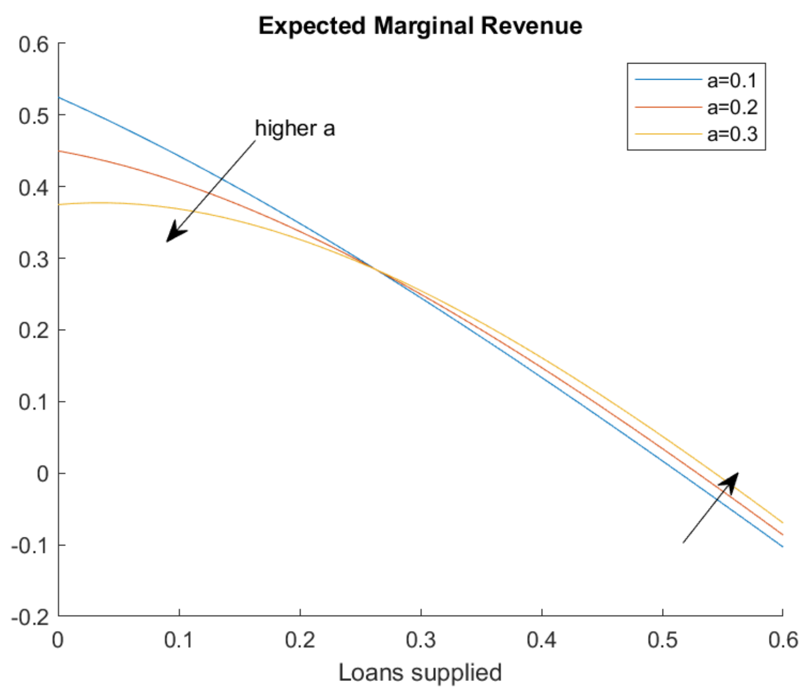
Under this more general function, we find that the affect of adverse selection on the lender's expected marginal revenue curve remains similar to the previous section. Figure (8) shows the expected marginal income curve of the lender for increasing levels of the adverse selection parameter (a_g) and the more general function. Comparing the bottom right panel of figure (5) with figure (8) shows that the figures are very similar. The key difference is that, under the more general function, the expected marginal revenue can be more curved. For completeness, figure (9) in the Appendix show the expected marginal revenue curve for different values of n.

[10]This feature can be seen in figure (7), where if all loans supplied is equal to 1, the average repayment is the same (in the example shown equal to 0.6) irrespective of n.

Due to the more general nature of the functional form, unfortunately, a closed-form characterization of the lender's optimal lending strategy is not possible. However, we can restate Lemmas 1 and 2 under certain parameters. Specifically, we find that the expected marginal revenue curve of the lender is more sensitive to increased adverse selection for most positive levels of loans supplied (formally written as lemma 4 in the Appendix, the counterpart of lemma 1). The lender's expected marginal revenue curve is greater for groups with less adverse selection at low values of supply (formally written as lemma 5 in the Appendix, the counterpart to Lemma 2).

In the previous section, we showed that—if two groups were identical except for their adverse selection parameter—the group with more adverse selection would have the largest rise in lending following a reduction in the lender's funding cost (Proposition 1). In this more general formulation, we can only state this result for certain parameters ((Proposition 2)).

Figure 8. The relationship between the expected marginal revenue curve and adverse selection (a) in the more general model



This figure shows the relationship between the expected marginal revenue curve and adverse selection in the more general model. The black arrows demonstrate how the curves change as we increase a_g. A higher a_g ensures that the expected marginal revenue curve is lower at low levels of loan supply and higher for high levels of loan supply.

Proposition 2. *Consider an interior solution and two consumer groups, g and g', that solely differ on the problem of adverse selection within the group.* ***Then, the group with the larger degree of adverse selection will have a larger increase in lending if the bank's funding cost falls and loans supplied are less than $\bar{D}$, where $\bar{D} = 1 - \left(2/(n+2)\right)^{1/n}$.***

Proposition 2 states that the group with higher adverse selection will have a larger rise in lending if lending is sufficiently low. Moreover, there are some parameters where we find the opposite result. That is, if both n and loans supplied are high (so the cost of lending is very low), the group with the larger degree of adverse selection may have a smaller increase in lending if the lender's funding cost falls. This result follows from when the degree at which adverse selection changes in the population, n, and loans supplied are high, the expected marginal revenue curve is more inelastic if adverse selection is higher (this result is formally shown in lemma (4) in the Appendix). The intuition for this result is that for large n and large a_g, the marginal gain from a small increase in lending for the average repayment rate ($\tilde{p}$) becomes increasingly small, which causes the expected marginal revenue curve to become more inelastic. This feature can be clearly seen in figure (6), when n is high–for example in the yellow curve–the probability of success curve barely changes at low levels of θ.

4. Conclusion

This paper presents a simple model of financial intermediation that formalizes how monetary expansion may have heterogeneous effects for different consumers depending on the extent of adverse selection within each consumer group. We model adverse selection by allowing groups to vary in the degree of dispersion of borrower quality within group. In a simple model with linear changes in adverse selection we show that groups with more adverse selection will have larger rises in credit following a monetary expansion.

Our model concentrates on the effects of lending on the extensive margin. However, to fully examine how consumer welfare is affected by adverse selection and a monetary policy expansion, one must consider the effects on total lending, that is, both the intensive and extensive margin effects. Given there are several contractual measures to limit the adverse effects of adverse selection that simultaneously restrict loan sizes (such as loan-to-value restrictions, debt-to-income restrictions, credit scores) we may find different results for lending on the intensive margin. Therefore, a logical area for further theoretical (and empirical) research is to examine the distributional effects of adverse selection on total lending, and explicitly examine the intensive margin effects.

5. Funding and/or Conflicts of interests/Competing interests

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Appendix

A1: Lemma. The optimal level of loans provided by the lender is in the range of 0 and 2/3.

Lemma 3. *The optimal level of loans supplied by the lender is in the range of 0 and 2/3.*

Proof. The lender will not supply negative loans therefore 0 is a natural lower bound on the minimum number of loans.

To prove that the lender will not supply more than 2/3, we can rearrange the equilibrium demand function. Recall that the equilibrium demand for consumer group g is the following:

$$D_g^*(a_g) = \begin{cases} \frac{1}{2} - \frac{b_g \kappa_g c}{2\bar{p}_g}, & \text{if } \pi_g(R_g^*, D_g^*) > 0 \ \& \ a_g = 0 \\ \frac{1}{3a_g}\left[2a_g - 2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p} - 6a_g b_g \kappa_g c}\right], & \text{if } \pi_g(R_g^*, D_g^*) > 0 \ \& \ a_g > 0 \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

If $a_g = 0$, then the upper bound on D*(0) is trivially 1/2.

Therefore, we need to show that the following condition holds for all positive values of a_g and $\bar{p}_g$:

$$D_g^*(a_g) = \frac{1}{3a_g}\left[2a_g - 2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p}_g - 6a_g b_g \kappa_g c}\right] \leq \frac{2}{3} \tag{20}$$

To start note that the equilibrium demand, D^*, is decreasing in the term $6a_g b_g \kappa_g c$. Therefore, to ascertain the upper bound on D^*, we can set this term to zero.

$$D_g^*(a_g) = \frac{1}{3a_g}\left[2a_g - 2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p}_g}\right] \leq \frac{2}{3} \tag{21}$$

$$= \frac{2}{3} + \frac{1}{3a_g}\left[-2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p}_g}\right] \leq \frac{2}{3} \tag{22}$$

Where to move from equation (21) to (22), we just rearranged the terms. Therefore, simplifying equation (22), we solely need to show the following:

$$-2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p}_g} \leq 0 \tag{23}$$

Which can be written as:

$$4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p}_g \leq 4\bar{p}_g^2 \tag{24}$$

Which simplifies to:

$$a_g(a_g - 2\bar{p}_g) \leq 0 \tag{25}$$

Inequality (25) holds because a_g is positive and a_g is restricted to be less than or equal to $\bar{p}_g$)). ☐

A2: Proof of proposition (1).

Proof. Proof of proposition (1).

To prove proposition (1), we will differentiate the equilibrium demand with respect to the lender's cost of funding, c.

Recall that the equilibrium demand and interest rate for consumer group g is the following:

$$D_g^*(a_g) = \begin{cases} \frac{1}{3a_g}\left[2a_g - 2\bar{p}_g + \sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p} - 6a_g b_g \kappa_g c}\right], & \text{if } \pi_g(R_g^*, D_g^*) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

Therefore, for an interior solution, differentiating the equilibrium demand with respect to c gives:

$$\frac{dD_g^*}{dc} = -\frac{b_g\kappa_g}{\left[\sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p} - 6a_g b_g \kappa_g c}\right]} \tag{27}$$

Recall that ΔD_g^* is defined as the change in equilibrium loans supplied to group g after a small fall in the lender's cost of lending and we have assumed that both groups have the same demand elasticities (b_g) and marginal cost of lending (κ_g). Then we can write:

$$\left[\Delta D_{g'}^* - \Delta D_g^*\right] = \frac{b_g\kappa_g}{\left[\sqrt{4\bar{p}_g^2 + a_{g'}^2 - 2a_{g'}\bar{p} - 6a_{g'} b_g \kappa_g c}\right]} - \frac{b_g\kappa_g}{\left[\sqrt{4\bar{p}_g^2 + a_g^2 - 2a_g\bar{p} - 6a_g b_g \kappa_g c}\right]}$$

Simplifying and rearranging this equation, we get:

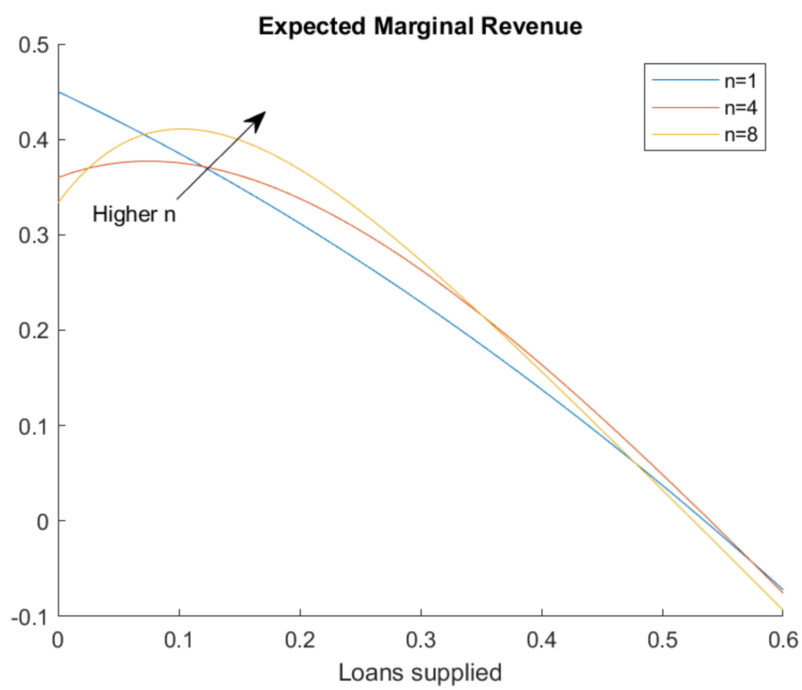
$$\text{sign}\left[\Delta D^*_{g'} - \Delta D^*_g\right] = \text{sign}\left[(a_g - a_{g'})(a_{g'} + a_g - 2\bar{p} - 6b_g\kappa_g c)\right] \quad (28)$$

$$= \text{sign}\left[2\bar{p} + 6b_g\kappa_g c - (a_{g'} + a_g)\right] \quad (29)$$

$$= \text{sign}[+] \quad (30)$$

Where equation (29) follows from the assumption that group g' has a higher degree of adverse selection that group g and equation (30) follows from $a_g < \bar{p}_g$ for all g. □

FIGURE 9. The relationship between the expected marginal revenue curve and the degree at which adverse selection in the population changes (n)



This figure shows the relationship between the expected marginal revenue curve and the loans supplied as we adjust the degree of how adverse selection changes within the population, n, holding all other parameters constant. The black arrow demonstrates how the curves change as we increase n. Recall that the average repayment probability increases relatively fast for increases in loan supply at low levels of loan supply, as we increase n. Therefore, we observe a similar effect when examining the lender's expected marginal revenue curve. That is, a higher n ensures that the expected marginal revenue curve can actually rise as we increase in loan supplied at low levels of loan supply. In other words, since the average repayment probability rises quickly for higher n, even though the lender is charging lower interest rates (to induce greater loan takeup), the lender's marginal (and consequently average) revenue can actually rise.

A3: Additional figures and proofs for the more general model.

Lemma 4. ***The lender's marginal revenue is more sensitive—more elastic—to changes in loans supplied if adverse selection (a_g) is higher for a low level of loans supplied.*** *Formally,*

$$\frac{d^2E[MR]}{da_g dD_g} \geq 0 \quad \forall D_g \in [0, D^e] \qquad (31)$$
$$\textit{where } D^e = 1 - \left(\tfrac{2}{3n+2+n^2}\right)^{1/n}$$

Proof. The lender's expected revenue curve in the more general model is:

$$E[\text{Revenue}] = \left(\mathrm{D}\,\overline{p} + \frac{a\left(\left(1-\mathrm{D}\right)^n - 1\right)\left(1-D\right)}{n+1}\right)(1-D) \qquad (32)$$

Differentiating the expected marginal revenue curve by D_g and a_g gives:

$$\frac{d^2E[MR]}{da_g dD_g} = \frac{(1-\mathrm{D})^n(3n+2+n^2)-2}{n+1} \qquad (33)$$

Rerranging equation (33) shows that:

$$\frac{d^2E[MR]}{da_g dD_g} \geq 0 \quad \text{if} \quad D_g \in [0, D^e]$$
$$\frac{d^2E[MR]}{da_g dD_g} \leq 0 \quad \text{if} \quad D_g \in [D^e, 1]$$
$$\text{where } D^e = 1 - \left(\frac{2}{3n+2+n^2}\right)^{1/n}$$

□

Lemma 5. ***The lender's marginal revenue curve is greater for groups with less adverse selection at low values of supply.*** *Consider two consumer groups, g and g', that have identical demand elasticities, average repayment probabilities but one group, group g', has a greater degree of adverse selection within the group (that is, $a_{g'}$ is strictly greater than a_g). Then the lender's marginal revenue curve for the group with a lower adverse selection is weakly greater for low supply and weakly lower for large supply.*

Proof. To demonstrate this result, we differentiate the marginal revenue curve with respect to the degree of adverse selection (a_g).

$$\frac{dE[MR]}{da_g} = -\frac{(1-D)\,(n+2)\,(1-\mathrm{D})^n - 2)}{n+1}$$

$$\frac{dE[MR]}{da_g} = \begin{cases} \leq 0, & \text{if } \ 0 \leq D_g \leq \bar{D} \\ \geq 0, & \text{if } \ D_g \geq \bar{D} \end{cases}$$

$$\textit{where} \qquad \bar{D} = 1 - \left(\frac{2}{n+2}\right)^{1/n}$$

$$\textit{and} \qquad 0 < \bar{D} < 1$$

□

Proof of Proposition (2).

Proof. Two additional initial results will help the exposition of this proof.

First, $\bar{D}$ is strictly less than D^e, where $D^e = 1 - \left(\frac{2}{3n+2+n^2}\right)^{1/n}$. Recall that D^e is the point such that the expected marginal revenue curve is more elastic for the group with higher adverse selection (lemma 4).

Assume that $\bar{D}$ is strictly less than D^e then:

$$D^e > \bar{D}$$

$$1 - \left(\frac{2}{3n+2+n^2}\right)^{1/n} > 1 - \left(\frac{2}{n+2}\right)^{1/n} \tag{34}$$

Rearranging equation (34), multiplying by the power n, and dividing by 2, we get:

$$n+2 < 3n+2+n^2 \tag{35}$$

And since n is greater than or equal to 1, equation (35) is true therefore $\bar{D}$ is strictly less than D^e.

Second, the second differential of the marginal revenue curve is negative (so the elasticity of the marginal revenue curve is decreasing as loans increase). To observe this result, we can just take the second derivative with respect to loans supplied:

$$\frac{d^2E[MR]}{dD_g^2} = -n(1-D)^{n-1}(2+n)a$$

$$\leq 0 \quad \forall D \in [0,1] \tag{36}$$

To prove Proposition (2), we will use lemmas (4) and (5) and the two results above.

From lemma (5) we know that lender's marginal revenue curve is lower for the group with more adverse selection if loans supplied are less than $\bar{D}$, where $\bar{D} = 1 - \left({}^{2}/_{n+2}\right)^{1/n}$. Therefore, for an interior solution, at a given level of lender funding cost, the lender will give more loans to the group with lower adverse selection.

We showed that $\bar{D}$ is strictly less than D^e. Therefore, if loans supplied are less than or equal to $\bar{D}$, we know that the lender's expected marginal revenue curve is more elastic for the group with higher adverse selection (from lemma 4).

Therefore, since the loans supplied are less than $\bar{D}$ and D^e for both groups, then we know that the group with more adverse selection curve is more elastic at the point that the marginal revenue curve for each group is equal to the lender's cost of funding. This final result follows from the second differential of the marginal revenue curve is negative (so the elasticity of the marginal revenue curve is decreasing as loans increase). Therefore, the group with more adverse selection will have a larger rise in lending for a small fall in the lender's cost of funding if lending for both groups is strictly less than $\bar{D}$.

□